

On Neutrino Emission From Dense Matter Containing Meson Condensates

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Abstract

We consider the rate at which energy is emitted by neutrinos from the dense interior of neutron stars containing a Bose condensate of pions or kaons. The rates obtained are larger, by a factor of 2, than those found earlier, and are consistent with those found for the direct Urca processes.

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It has been suggested that matter at several times the density of normal nuclei, as found in the cores of collapsing massive stars or in the dense interior of neutron stars, may contain a Bose condensate of pions [1] or kaons [2]. A number of aspects of meson condensation under such conditions have been explored, one being the possible role played by condensates in the thermal evolution of neutron stars. Neutron stars are born with internal thermal energies of some tens of MeV. In the first $10^5 - 10^6$ years after formation, the chief mechanism for energy loss is the emission of neutrinos and anti-neutrinos from matter in the interior of the neutron star. Which neutrino emitting reactions are most effective in removing energy from the neutron star is the subject of numerous studies. (For a review, see Ref. [3].) In 1965, Bahcall and Wolf [4] showed that if pions were found in the interior of neutron stars, the neutrino emission rate could be considerably higher than that found in the absence of pions. Following the suggestion that attractive p-wave interactions could lead to pion condensation [1], Maxwell, Brown, Campbell, Dashen and Manassah [5] (MBCDM), computed the rate of neutrino energy emission from matter containing a pion condensate. Shortly after the idea arose that kaons could condense in matter due to attractive s-wave interactions [2], Brown, Kubodera, Page and Pizzochero [6] and Tatsumi [7] pointed out that kaon condensation could also lead to rapid cooling, by a reaction analogous to that considered for pions.

In this note, we point out what we believe is an error in the emission rate originally evaluated by MBCDM and in nearly all subsequent studies. The corrected emission rates are consistent with those obtained recently in a separate study of a closely related emission process, the direct Urca process [8]. Correcting this error is expected to have little effect on the general discussion of the relative importance of meson induced cooling processes in relation to other processes considered viable [3,9].

Briefly, the discrepancy between the rates found in the presence of meson condensates on the one hand [5–7,11] and the direct Urca process [8], on the other, is apparent by observing that the net rate of emission from matter with a meson condensate should, in the limit of a vanishing condensate, reduce to the rate for the direct Urca process for nucleons. That it

does not may be seen as follows. Consider the reactions

$$n(\pi) \rightarrow n(\pi) + e^- + \bar{\nu}_e \text{ and } n(\pi) + e^- \rightarrow n(\pi) + \nu_e , \quad (1)$$

treated in MBCDM. In Eq. (1), $n(\pi)$ denotes an excitation which is a superposition of a neutron and a proton, and reduces to a free neutron in the absence of a pion condensate. The pion condensate is parametrized by a momentum k , and an angle θ describing the strength of the condensate, or the degree to which the pion is chirally rotated away from its value in the vacuum by the presence of matter [1]. In the case of kaon condensation, the condensate is spatially uniform, characterized only by an angle θ , describing the degree of V-spin rotation of the kaonic ground state [2]. Several reactions have been considered in the presence of a kaon condensate, and are listed in Table 1. The pairs of reactions (1) and (2) of Table 1 are analogous to reactions (1) above, while reactions (3) of Table 1, to be described below, are related to the direct Urca process. In all cases, the rate of energy emission due to neutrinos is given by the square of the relevant matrix element \mathcal{M} , summed over over spin states of all participants in the reaction:

$$|\mathcal{M}|^2 = H^{\mu\nu} L_{\mu\nu} , \quad (2)$$

where $H^{\mu\nu}$ and $L_{\mu\nu}$, in covariant form, are the hadronic and leptonic contributions, respectively. It is easy to check that Refs. [5–7,11] are internally consistent in the sense that differences reside only in the hadronic contribution $H^{\mu\nu}$, and that the overall coefficient in the emission rate is consistent with the final result of MBCDM, Eq. (53). (The actual evaluation of emission rates requires rather involved phase space integration and will not be carried out here. For details, we refer to the original references [5–7]. We have checked that phase space integrals have been correctly performed in the references quoted. Therefore, we confine the discussion below to matrix elements only.) Furthermore, by taking the $\theta \rightarrow 0$ limit in Eq. (4.15) of ref. [11], the rate obtained for reaction (3) of Table 1, one obtains a prediction for the energy emission rate due to the neutron direct Urca process. The emission rate for the direct Urca process thus obtained is a factor of 2 smaller than the recent result

of Ref. [8], a non-relativistic calculation that is easy to check against the rate of neutron beta decay in free space.

The direct Urca process for nucleons refers collectively to neutron beta decay

$$n \rightarrow p + e^- + \bar{\nu}_e \quad , \quad (3)$$

and electron capture on protons

$$p + e^- \rightarrow n + \nu_e \quad . \quad (4)$$

For many years, it was thought that the direct Urca process was forbidden at low temperatures, momentum conservation never being possible due to the relatively small proton and electron Fermi momenta in relation to that of the neutrons. However, Lattimer, Pethick, Prakash and Haensel [8] (see also Boguta [12]), recently drew attention to the fact that in several recent models of nuclear matter at high density, the concentration of protons may indeed be so large that these kinematical conditions may be met. This could lead to very rapid cooling of neutron stars, with cooling rates even higher than those found in the presence of meson condensates.

The origin of the factor of 2 discussed above, which we believe is an error arising in the original work of MBCDM, is not immediately clear. One possible explanation is simply that the overall numerical coefficient in the final emission rate, Eq. (53), was underestimated. (Note that MBCDM also stress the importance of maintaining consistency with the neutron beta decay rate.) However, we believe the problem already arises at an earlier stage of the calculation. Preceding Eq. (29) of MBCDM it is stated that the relevant matrix element, $|\mathcal{M}|^2$, includes sums over spins of all participants in the reaction. The leptonic contribution $L^{\mu\nu}$ is obtained from Eq. (146.3) of Lifshitz and Pitaevskii [13]. However, the $L^{\mu\nu}$ of Ref. [13] is expressed in terms of density matrices, which do not include a sum over the spins of the electrons in the final state, and would therefore seem to be inconsistent with the definition preceding Eq. (29) of MBCDM. Summing over the spin states gives the conventional electron projection operator, which is a factor of 2 larger than the density matrix. We therefore expect

the leptonic tensor, relevant for the emission of an electron of four-momentum p_e , and an antineutrino of four-momentum, p_ν , to read

$$L^{\mu\alpha} = 8 (p_e^\mu p_\nu^\alpha + p_e^\alpha p_\nu^\mu - (p_e \cdot p_\nu) g^{\mu\alpha} - i \varepsilon^{\mu\alpha\gamma\delta} (p_e)_\gamma (p_\nu)_\delta) , \quad (5)$$

which is a factor of 2 larger than the corresponding expression given in Eq. (30) of MBCDM. Conversely, if we choose to use the density matrix formulation, we need to sum over the final spins of the electrons, as in the example of Ref. [13], thereby obtaining an additional factor of 2 over that given in MBCDM.

In what follows, we summarize the various emission rates discussed in the literature with the required increase by a factor of 2. It is most convenient to express the results in terms of the emission rate for the closely related direct Urca process, Eqs.(3) and (4). The rate at which energy is emitted per unit volume by reactions (3) and (4), in matter in which all the participating fermions are degenerate, is [8]

$$\begin{aligned} \dot{E}_{Urc} &= \frac{457\pi}{10080} \frac{G_F^2 \cos^2 \theta_C (1 + 3g_A^2)}{\hbar^{10} c^5} m_n m_p \mu_e (k_B T)^6 \\ &= 2.21 \times 10^{26} \left(\frac{\mu_e}{100 \text{ MeV}} \right) (1 + 3g_A^2) \cos^2 \theta_C T_9^6 \text{ erg cm}^{-3} \text{ sec}^{-1} . \end{aligned} \quad (6)$$

Here, $G_F = 1.436 \times 10^{-49} \text{ erg cm}^3$ is the Fermi coupling constant, $\theta_C \simeq 0.223$ is the Cabibbo angle, $g_A = 1.26$ is the nucleon axial-vector coupling constant, m_n is the neutron mass, m_p is the proton mass, μ_e is the electron chemical potential, and T is the temperature, T_9 being its value in units of 10^9 K . In Eq. (6), we have neglected the in-medium modification of the weak-interaction matrix elements and the effects of possible superfluidity of neutrons and superconductivity of protons [14]. The energy emission rate is $2\dot{E}_{Urc}$ if the muon Urca process can also occur.

The energy emission rate of the reactions in Eq. (1) is, for small values of θ (see MBCDM),

$$\dot{E}_\pi = g \left(\frac{p_e}{k}, \frac{p_n}{k} \right) \frac{\theta^2}{4} \left[1 + \left(\frac{g_A k}{p_e} \right)^2 \right] \dot{E}_{Urc} , \quad (7)$$

where p_e and p_n are, respectively, the electron and neutron Fermi momenta, and

$$g \left(\frac{p_e}{k}, \frac{p_n}{k} \right) = -\frac{1}{8} \sum (-1)^{n-} \left| \pm 1 \pm \frac{p_e}{k} \pm \frac{p_n}{k} \pm \frac{p_n}{k} \right| . \quad (8)$$

In Eq. (8), the sum is over the 16 ways of assigning signs in the expression in the modulus, and n_- is the number of negative signs in a particular term. The function g goes to unity in the limit $k \rightarrow 0$. When p_e is less than the length of any momentum vector that can be made up from k and two momenta with magnitudes equal to the neutron Fermi momenta, $g = p_e/k$ [5].

In connection with pion condensation, we remark that if the proton concentration is sufficiently large, it will be possible for the proton analogs of the processes (1) to occur. To the best of our knowledge, this process has not been considered previously. For this process to be allowed, the sum of two proton Fermi momenta and the electron Fermi momentum must exceed k . If this process occurs, its rate will be comparable to that of the neutron process (1).

In Table 1, we list the energy emission rate, in terms of \dot{E}_{Urc} , for the possible reactions in the presence of a kaon condensate described earlier.¹ It is worth emphasizing that the reactions discussed above take place only if energy-momentum conservation can be satisfied among the participants for the temperatures and densities under consideration. In most models describing conditions in the interior of cooling neutron stars, energy-momentum conservation is rather easily met in reactions (1) and (2) of Table 1 (an important factor in their being considered as a viable cooling agent [5–7]), whereas reactions (3) of Table 1 take place only if the concentration of protons in the matter is sufficiently large [10,11].

In conclusion, we believe that we have resolved a discrepancy between the published neutrino energy emission rates from dense matter with condensates, on the one hand [5–7,11], and that found for the direct Urca process [8], on the other. Rates found earlier in the presence of a condensate are correct if multiplied by a factor of 2. (The recent Ref. [10] contains this correction.) We do not expect this correction to significantly alter our current

¹ The treatment of the axial current matrix elements here is consistent with that of Ref. [6], and relates to that of Refs. [7,11] by the replacement $g_A \rightarrow 1$ in the matrix element of A_g^μ .

understanding of the thermal evolution of neutron stars [9].

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TABLES

TABLE I. Neutrino emissivity in the presence of a charged kaon condensate in terms of \dot{E}_{Ura} , the neutrino emissivity for the direct Urca process, Eq. (5). Process 1 is the one considered in the original work of Brown, Kubodera, Page and Pizzochero [6] and Tatsumi [7] .

	Reaction	\dot{E}/\dot{E}_{Ura}
1	$n(K) \rightarrow n(K) + e^- + \bar{\nu}_e$	
	$n(K) + e^- \rightarrow n(K) + \nu_e$	$\frac{1}{4} \sin^2 \theta \tan^2 \theta_C$
2	$p(K) \rightarrow p(K) + e^- + \bar{\nu}_e$	
	$p(K) + e^- \rightarrow p(K) + \nu_e$	$\sin^2 \theta \tan^2 \theta_C$
3	$n(K) \leftrightarrow p(K) + e^- + \bar{\nu}_e$	
	$p(K) + e^- \rightarrow n(K) + \nu_e$	$\cos^2(\theta/2)$